Mid-term Exam

Question 1: A particle moves in a potential

$$V(x) = \begin{cases} \infty & if\ x > 0 \\ ax & if\ x \leq 0 \end{cases}$$

(i) Suggest a variational ansatz for the ground state wavefunction and calculate an estimate of its energy.
(ii) Estimate the expectation value of the position.

Question 2: A particle moves in a potential

$$V(x) = \begin{cases} V_0 & if\ -b \leq x \leq b \\ 0 & if\ b < |x| \leq a \\ \infty & if\ |x| > a \end{cases}$$

(i) Calculate the eigenfunctions of the particle in the case $V_0 = 0$.
(ii) Use perturbation theory to calculate the changes in the energies of the two lowest energy states of the system to lowest order in V_0 .

Question 3: Two non-interacting spin-1/2 particles are placed in a 1D potential

$$V(x) = \begin{cases} 0 & if\ 0 < x < L \\ \infty & otherwise \end{cases}$$

A strong magnetic field is applied so that the projection of the total spin on the z-axis is maximized. Write down the ground state wavefunction.

Question 4: Consider a rigid rotator such as a CO molecule. Ignoring the center-of-mass motion, the Hamiltonian for the molecule is given by $\hat{H} = \frac{1}{2I} \hat{L}^2$, where I is the moment of inertia and $\hat{L}$ represents the magnitude squared of the angular momentum.

i) Determine an expression for the energy eigenvalues for the rigid rotator.
ii) List *all* the energy eigenstates for the two lowest energy eigenvalues.
iii) Now consider the same rigid rotator placed in a constant magnetic field $B = B_0 z$ **,** the Hamiltonian is $\hat{H} = \frac{1}{(2I)} \hat{L}^2 + \omega_0 \hat{L}_z$ where $\omega_0 > 0$ is a constant which depends on B_0 . Verify that the *eigenstates* are the same as those for $\omega = 0$ case and determine the *eigenvalues*. List the four lowest values assuming that $\omega\hbar < 2\hbar^2 / 3I$.

March 18, 2008